

Close Wed: HW_2A, 2B, 2C (5.3,5.4,5.5)

Please visit office hours:

1:30-3:00pm in Comm. B-006

5.5 The Substitution Rule

Entry Task (Motivation):

1. Find the following derivatives

Function	Derivative?
$\cos(x^2)$	
$\sin(x^4)$	
$e^{\tan(x)}$	
$(\ln(x))^3$	
$\ln(x^4 + 1)$	

2. Rewrite as integrals:

$$\int dx = \cos(x^2) + C$$

$$\int dx = \sin(x^4) + C$$

$$\int dx = e^{\tan(x)} + C$$

$$\int dx = (\ln(x))^3 + C$$

$$\int dx = \ln(x^4 + 1) + C$$

3. Guess and check the answer to:

$$\int 7x^6 \sin(x^7) dx =$$

Observations:

1. We are reversing the “chain rule”.
2. In each case, we see
“inside” = a function inside another
“outside” = derivative of inside

To help us mechanically see these connections, we use what we call:

The Substitution Rule:

If we write $u = g(x)$ and $du = g'(x) dx$,
then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Aside (you do not need to write this)

Some theory

Recall:

$$\int_a^b f(g(x))g'(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(g(x_i))g'(x_i)\Delta x$$

If we replace $u = g(x)$, then we are “transforming” the problem from one involving x and y to one with u and y .

This changes **everything** in the set up. The lower bound, upper bound, widths, and integrand all change!

Recall from Math 124 that

$$g'(x) = \frac{du}{dx} \approx \frac{\Delta u}{\Delta x}$$

(with more accuracy when Δx is small)

Thus, we can say that

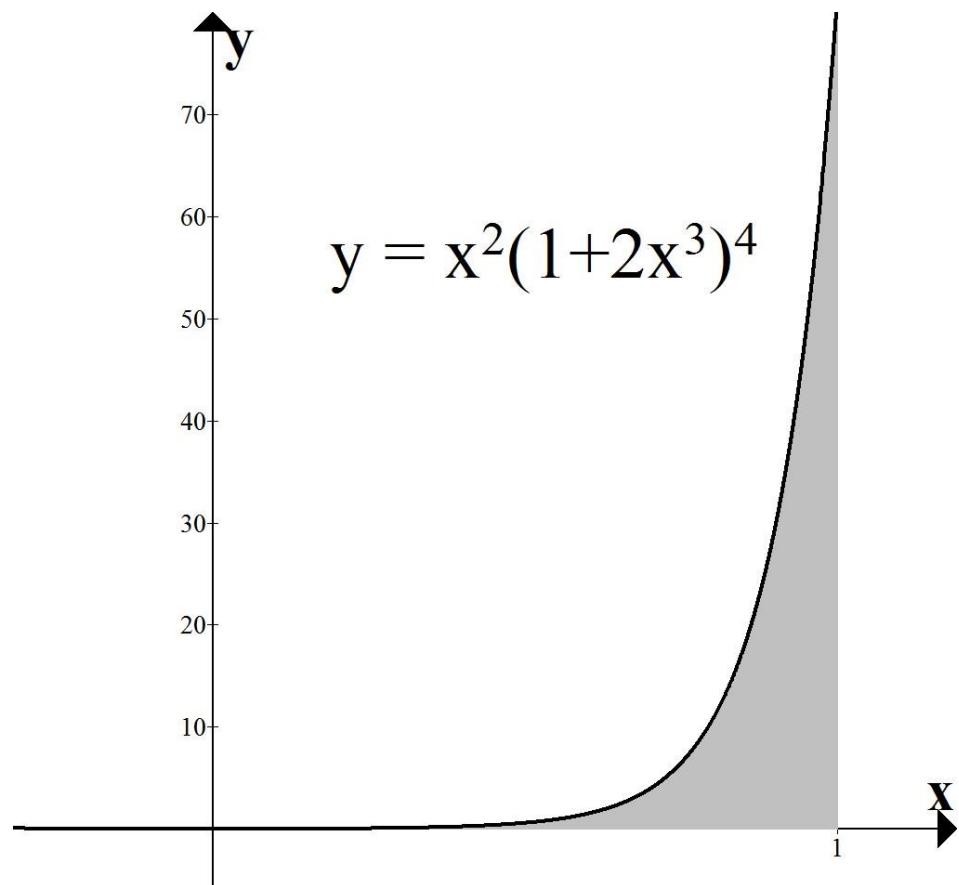
$$g'(x)\Delta x \approx \Delta u$$

In other words, if the width of the rectangles using x and y is Δx , then the width of the rectangles using u and y is $g'(x)\Delta x$.

And if we write $u_i = g(x_i)$, then

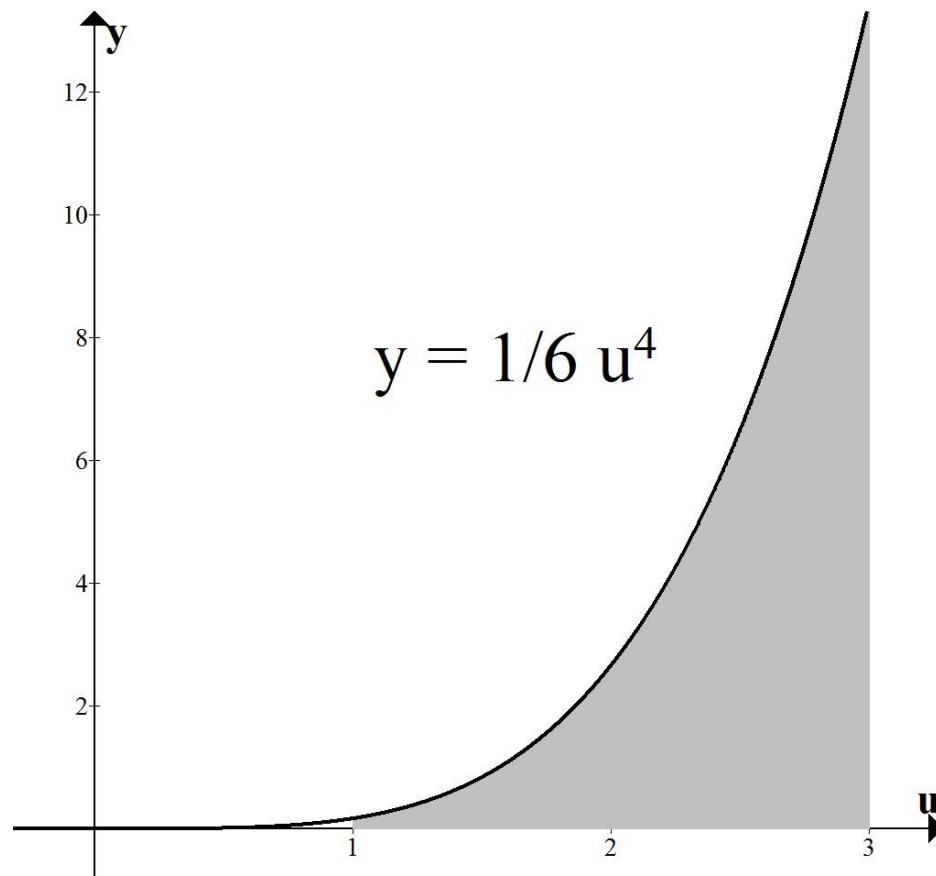
$$\begin{aligned} \int_a^b f(g(x))g'(x)dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(g(x_i))g'(x_i)\Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(u_i)\Delta u \\ &= \int_{g(a)}^{g(b)} f(u)du \end{aligned}$$

Here is a visual example of this transformation



$$\int_0^1 x^2(1+2x^3)^4 dx$$

Using $u = 1 + 2x^3$ and $du = 6x^2 dx$, we get



$$\int_1^3 \frac{1}{6} u^4 du$$